On Twin Amicable Numbers

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"Perfect numbers, like perfect men, are very rare." (Descartes)

Abstract

The aim of this short paper is to examine twin amicable numbers (see: [A273259]) using a modified analogy of the twin primes and to present some numerical observations both on the twin amicable numbers and on the degree of relationship between them.

Introduction: Amicable Numbers and Twin Primes

An amicable pair (m,n) contains two integers. The sum of the divisors of m is equal to n and the sum of the divisors of n is equal to m, so s(m)=n; s(n)=m where s(n)=sigma(n)-n (the list of divisors contains 1 as a divisor, but does not contain the number itself). m is necessarily abundant (s(n)>n) and n is deficient (s(n)<n) [1].

The examination of the features of the amicable numbers has a long history, which begins with the ancient Greeks and Euler, but the research focused mainly on the relations between the amicable numbers and their divisors until now, since the amicable numbers are division based phenomena. We attempt to interpret some features of the amicable numbers from a different point of view, focusing on other relations between the amicable pairs.

Twin primes are prime pairs with p; p+2 form: there is not any number (except for an even p+1 number) between them. This field is a subject of intensive research and in this case the question is the relationship of the twin primes to each other [2].

Definitions: Twin Amicable Numbers and the Degree of Relationship

The concept of twin primes can be applied to the amicable numbers with some modifications. According to our definition, an amicable m,n pair (m < n) is twin if there is not any part of another amicable number between m and n. This logic makes possible to introduce the degree of relationship marked by rel: rel=0 if the amicable pair is a twin pair (m1<n1<m2<n2 or m1<m2<n2<n1).

In the case of twin amicable pairs**,** the value ranges of the two amicable pairs do not overlap each other: n1 < m2 and rel=0. For example: m1=220 and n1=284; m2=1184 and n2=1210:

m1 → n1

m2 → n2

According to this definition, rel=1 if there is only one part of another amicable pair (m2,n2) between m1 and n1 (n1<m2 or: m1<n2<n1), for example m1=67,095 and n1=71,145; m2=69,615 and n2=87,633:

m1 → n1

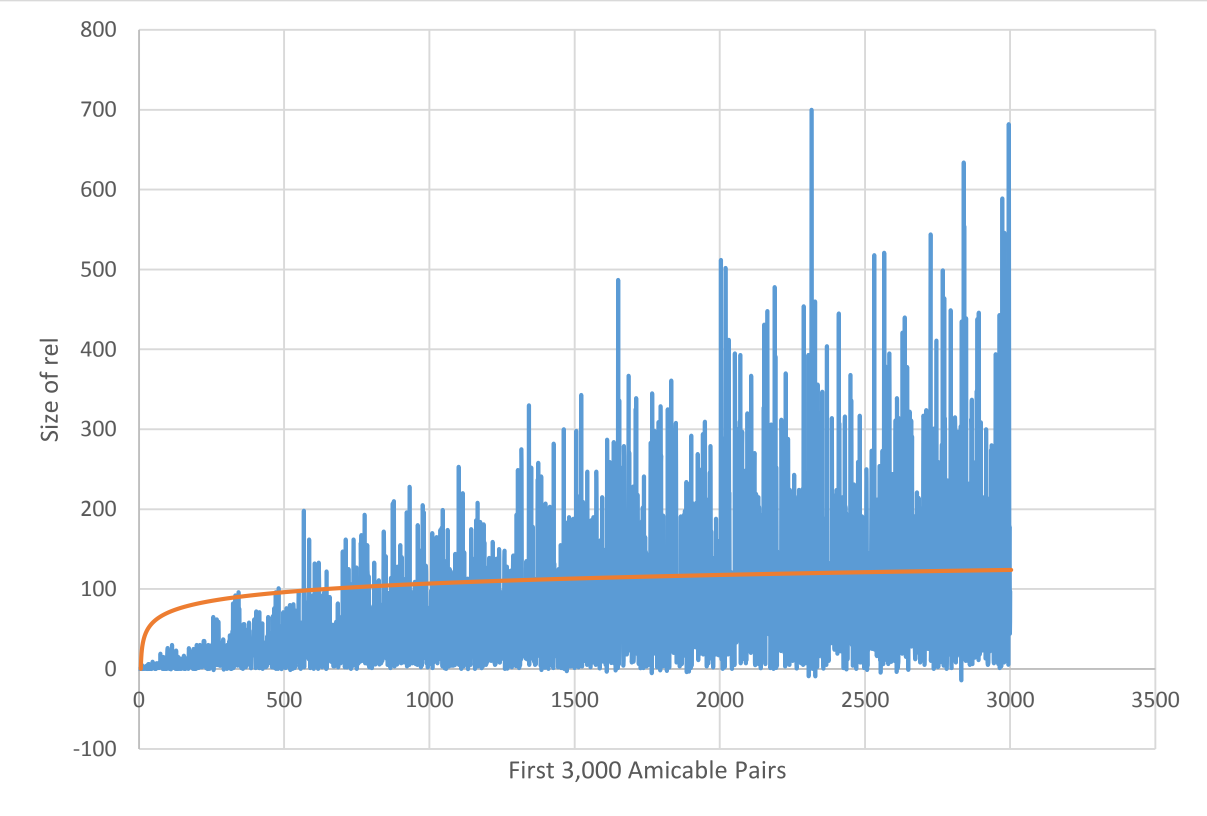
m2 → n2

rel=2 is 2 if there are two parts of another amicable pairs between m1 and n1; and so on.

Twin Amicable Numbers

The first seven amicable pairs are twin pairs (so they can be regarded as an octuple sequence). But this is misleading since the twin amicable pairs are the exceptions: examining the first

3,000 amicable pairs (up to 78,827,458,256), one can find only 75 twin amicable pairs (the biggest pair is: m= 64,112,960,650; n= 64,128,831,350. For the list of the known twin amicable pairs, see APPENDIX A).
Following the octuple sequence of twin amicable pairs, the ninth pair (m=63,020; n=76,084), is the first non-twin amicable pair (with rel=5) [3], [4].
It is known that the density of amicable pairs is asymptotically 0 [5]. Obviously, there are fewer twin amicable numbers than normal amicable pairs and their asymptotic density is 0, as well.



Some Open Questions

Even if it were known that there are infinitely many amicable numbers, this would not guarantee automatically that there are infinitely many twin or multiple twin pairs. What is more, it is not certain that there is a twin amicable pair not listed in APPENDIX A and it is an open question, too, whether there are other triple, quadruple etc. twin amicable pairs (see APPENDIX C).
The degree of relationship (rel) of all of the amicable pairs for the first 3,000 amicable pairs is shown on the above graph. The biggest value of rel up is 700 (m= 35,049,418,250; n= 54,192,685,558) and the size of rel seems to be growing following a roughly logarithmic rule [6].

APPENDIX A: Twin Amicable Pairs of the First 3,000 Amicable Pairs

	m	n
1	220	284
2	1184	1210
3	2620	2924
4	5020	5564
5	6232	6368
6	10744	10856
7	12285	14595
8	17296	18416
9	66928	66992
10	122368	123152
11	196724	202444
12	437456	455344
13	469028	486178
14	503056	514736
15	522405	525915
16	643336	652664
17	802725	863835
18	998104	1043096
19	1077890	1099390
20	1511930	1598470
21	1798875	1870245
22	2082464	2090656
23	2236570	2429030
24	5459176	5495264
25	6377175	6680025
26	6993610	7158710
27	7677248	7684672
28	8262136	8369864
29	9363584	9437056
30	10254970	10273670
31	13921528	13985672
32	15002464	15334304
33	16137628	16150628
34	17908064	18017056
35	20308995	20955645
36	25596544	25640096
37	26090325	26138475
38	28118032	28128368
39	28608424	29603576
40	34364912	34380688
41	44139856	44916944
42	46991890	48471470
43	48641584	48852176

44	50997596	51737764
45	96304845	96747315
46	109410345	110132055
47	133178325	133471275
48	175032884	175826716
49	195857415	196214265
50	209309704	209816696
51	216392216	218772184
52	287250632	287551288
53	426386025	426684375
54	573216416	576387424
55	595858064	596654896
56	637756665	639580935
57	766292835	766512285
58	902335744	903709952
59	1110676384	1111963616
60	1786492785	1790052495
61	1930301618	1930741582
62	4429428675	4436670525
63	5113096443	5113841733
64	5668081984	5675159744
65	7074650624	7076729344
66	9490622048	9500349952
67	10652028345	10654626375
68	21226267876	21245207324
69	24919600064	24930122944
70	25937232896	25941935104
71	30575568896	30594296224
72	42737732650	42756795350
73	47406476608	47436700736
74	61764442005	61788940395
75	64112960650	64128831350

APPENDIX B: Double, Triple, etc. Twin Amicable Pairs

Notice that these amicable pairs are listed as twin amicable numbers above, as well:

Double twin amicable pairs:

	m	n
1	998104	1043096
2	1077890	1099390

Triple twin amicable pairs:

	m	n
1	1798875	1870245
2	2082464	2090656
3	2236570	2429030
1	26090325	26138475
2	28118032	28128368
3	28608424	29603576

Quadruple amicable pairs:

	m	n
1	437456	455344
2	469028	486178
3	503056	514736
4	522405	525915

Octuple amicable pairs:

	m	n
1	220	284
2	1184	1210
3	2620	2924
4	5020	5564
5	6232	6368
6	10744	10856
7	12285	14595
8	17296	18416

Notes

[1] Weisstein, Eric W. "Amicable Pair." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/AmicablePair.html

[2] Weisstein, Eric W. "Twin Primes." From MathWorld--A Wolfram Web Resource. http://mathworld.wolfram.com/TwinPrimes.html

[3] Data based on the list of amicable numbers from http://djm.cc/amicable2.txt . Note that we examined the degree of relationship only for the first 3,000 pairs. The reason: it is possible that we would fail to notice a sufficiently large amicable pair’s smaller part (m) calculating the degree of relationship, since we do not know the larger part (n).

[4] For numerical calculations, a software was used written by Béla Galántai (Hungary)
[5] Pomerance, Carl: On the distribution of amicable numbers II., 1980.
https://math.dartmouth.edu/~carlp/Amicable2.pdf
[6] Calculating the estimated upper quartile (=75% quartile), rel_{upq}=15.494 ln(x).

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