Comparison of Vibration Spectra and Cepstra used for Determining the Rotational Speed of Rotary Machines

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Abstract — Sufficient accurate estimation of rotational speed of rotary machines usually has got high priority in technical applications. This information should be calculated for many diagnostic algorithms, control or regulation processes. Incorrectly estimated values could occur serious disturbances in the operation of machines. Additional instrumentation often may be obstructed due to lack of space, but the construct of the machine may also affect the accuracy of measurement. In such cases, vibration diagnostic tools can be the disposal of difficulty. Fixing of an acceleration sensor onto surface of measured device is not a major challenge. Furthermore we usually get similar results if the vibration was measured at several points. In most cases the sampled time signal results insufficient conclusions. The calculated spectra and cepstra can help us to determine the rotational speed more easily and more accurate. This paper presents the comparison of these two methods in terms of their usability and rotational speed estimation accuracy. A possible error of traditional optical measurement due to misalignment and benefits of the two other methods are illustrated in this article via measured data series of a DC motor driven system.

Index Terms — cepstrum, diagnostic, rotary machine, rotational speed, spectrum

I. INTRODUCTION

THIS paper is concerned with the development of methods to calculate rotational speed of rotary machines using encoder or vibration signals. Rotary machines are an integral part of our everyday life, even if we do not perceive it in all cases directly. But if we think about it: rotating machines are in our vehicles, in almost all of household appliances, children's toys, modern robots, production equipment, and even the vibration function of phones is performed by miniature rotary machines as well. It could be often a critically important task, to define an accurate measurement based method for estimating the operational state of the examined device. There are several speed measuring solutions to solve this problem. We can use easily compact encoder, combination of axle mounted holey wheel and optical sensor, tachometer, Hall effect sensor, etc. The question may arise, how should we measure the basic parameters of an operating machine, particularly the rotating speed, if the engineer have not taken care of it in the early design phase? Mounting sensors into a device subsequently is usually not an easy task. Especially if we have to modify a rotating component in this way, which is hidden well from users. The difficulty is mainly due to the compactness of modern devices. Axes are usually not directly accessible and there is insufficient free space near to the drive. In these cases vibration diagnostics could be the best alternative solution. [1][2][3][4][5]

II. VIBRATION MEASUREMENT OF ROTARY MACHINES

Mounting accelerometers onto the device surface takes not a major challenge. Furthermore there are non-contact vibration velocimeters, which have got almost no installation requirements. Nowadays in most applications piezoelectric accelerometers are used for vibration measurements because of their small physical size, the good sensitivity, the specially wide dynamic range, and probably the main reason of this choice could be, that they do not require reference. Such a sensor can be used without fixing it onto the surface via special probe-tip as well. In case of measurements which requires more accuracy magnetic fastening or using stud mounting is recommended. If the surface is unsuitable for these type of fixings, wax mounting can be also a feasible solution to the problem. Further information about accelerometers and their mounting can be found in [6].

A. Signal analysis

The rotational speed can be easily obtained from vibration signals that was sampled in time. In addition it is also possible to get detailed information about condition of the measured system. Via a bit more complex diagnostic the date of rotary machine breakdown can be well predicted as well. We can sort diagnostic algorithms into several categories: time domain analysis, frequency domain analysis, higher order spectral analysis, order analysis, wavelet analysis and many kind of joint domain analysis as time-spectra or time-cepstra representations. In this article the accuracy of estimated

^{01.05.2015.}

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rotational speed calculated via methods in time and frequency domain and the cepstra are analyzed.

B. The analyzed system configuration

The test system consists a Faulhaber 3557-K024-CS DC servo drive connected to a HB-20M-2 hysteresis braking motor. A wheel with radial oriented black and white stripes is fixed to the end of the driven shaft of assembly. Due this striped wheel the angular movement can be detected with a single reflective optical sensor. For vibration measurement a PCB 356A33 3D piezoelectric accelerometer was fixed onto the housing of the DC servomotor. This sensor was fixed to the surface with special beeswax.



Fig. 1. Optical rotational speed measurement (left) and vibration measurement (right) of the analyzed system configuration

III. POSSIBLE ERROR OF OPTICAL ROTATIONAL SPEED MEASUREMENT

Considering the layout (*Fig. 1.*), it becomes clear, that the mounting of an accelerometer takes much less time and effort as other alternatives. The advantages of an axle mounted striped wheel are that this way the rotational speed can be obtained with primitive signal processing methods with relatively good accuracy. Due to the mechanical structure – if the bearing is not suitable, or the system leaves the proposed operational range – intensive vibration can be observed at the natural frequencies of the analyzed system. Because of this phenomenon gross error of measurement can be occurred. If the striped wheel leaves the operational range – there will be undetectable stripes. The reason of the gross error is this loss

of information. In such cases we have to make conclusions based on the wrong measurement data. Of course there are several correction methods to repair the calculated result. However the result after the correction will never be so accurate, as it would be after an errorless measurement because of the loss of measured information.

Fig. 2. shows the fickleness of optical method above 2500 rev/min rotational speed due to the non-detected stripes of the wheel. In this presented case the fault caused up to 30% relative error of the result.

In the following a steady state measurement result will be interpreted which was taken in this high-speed range. The next figure (*Fig. 3.*) shows





Fig. 3. Measured signals of optical sensor (yellow) and the 3D accelerometer (blue, red, green) at steady state of the system

the measured acceleration and the compared voltage signal of the optical encoder as a function of time. The Figure shows clearly that there are cyclically missing signal changes at a particular angular range. In this case we can make correction, but speed estimation based on vibration is also possible. [1]

IV. ROTATIONAL SPEED ESTIMATION BY TIME SIGNAL PROCESSING

A. Comparison method of filtered time signal

A special beat diagram can be obtained from the sampled time signal. The source of cyclical changing vibration and the orientation of accelerometer must be taken into account at this calculation method. The calculated result describes the rotational speed.

For the beat localization in time we have to perform signal processing in two main steps. Firstly smoothing with a MA filter (Moving Average) (Eq. 1.) or RMS calculation (Root Mean Square) (Eq. 2.). Secondly comparison to a specified threshold level (Eq. 3.). In the following equations x is the input data, y is the output data and N marks the used window size. TH represents the threshold level used for comparison.

$$y_{MA}[n] = \frac{1}{N} \sum_{i=0}^{N-1} x[n-i]$$
(1)

$$y_{RMS}[n] = \sqrt{\frac{1}{N} \sum_{i=0}^{N-1} x[n-i]^2}$$
(2)



The calculated results for the second channel of the analyzed measurement (vertical acceleration) is shown in the following figure (Fig. 4.). In this way we get a result that is similar to the optical encoder output. The disadvantage of this method is the low resolution. Usually there is only one or two detectable threshold crossing per revolution, so the speed can be determined less often. In addition, this method do not produce any information about the possible fluctuation within one revolution.

In Fig. 4. every second threshold crossing indicates a full period of operation. Can be seen that a full revolution takes approximately 13 to 14 ms time that equals to 4500 rev/min.

B. Using Autocorrelation function for determining rotational speed

The ACF (Auto Correlation Function) shows the averaged repetition of the analyzed section of the recorded data series. This repeated phenomenon may come from several sources. The source can be an unbalanced part of the assembly, cyclically increased dissipative effect - such as bearing friction - or the uneven torque of the drive motor. All of these reasons produce periodic deviation in the measurable vibration acceleration.

The correlation is a mathematical tool for finding repeating patterns, such as the presence of periodic signal obscured by random noise. ACF is the correlation of a signal with itself at different points in time. It shows how similar is the analyzed data series to itself and gives information about period time of similarity. Prominent feature of ACF is that calculation for

> periodic signal results periodic output. By contrast the uncorrelated components such noises are eliminated by the transformation.

ACF can be computed by the following equation:

$$R_{XX}(\tau) = \int_{-\infty}^{\infty} x(t)x(t-\tau)dt$$





0,07

0,08

0,09

Fig. 5. ACF of acceleration

0,01

0,02

0,03

0,04

Acceleration [g]

Main properties of the result of (4) are:

•
$$R_{XX}(\tau) = R_{XX}(-\tau)$$
 (5)
The function is symmetric to the origin.

•
$$R_{XX}(0) = \int v^2 dt$$
 (6)

The calculated value in 0 is proportional to the signal energy.

•
$$R_{XX}(\tau) = \int_{-\infty}^{\infty} x(t)x(t-\tau)dt$$
(7)

Global maximum of the function is always in 0.

Previous equations are defined for continuous time signal, however we usually have sampled measurement data series in discrete time domain. That's why modifying (4) is needed. For ACF calculation in case of discrete time signals the following equation can be used:

$$y_{ACF}[n] = \sum_{k=0}^{N-1} x[k] \cdot x[n+k]$$
(8)

Fig. 5 shows the ACF calculated for the vertical vibration acceleration of the analyzed system. The second prominent peak of the function after the zero location indicates the period time of the operation. An important property of this method is that it produces the averaged rotational speed for the related time section. This property causes that this method is effective only at steady states of the examined system. A prominent advantage against the previously presented time based method is that the result is much less influenced by transient deviations and random noises.

V. ROTATIONAL SPEED ESTIMATION IN FREQUENCY DOMAIN

During the traditional spectral analysis, we study how a signal's power is distributed in the frequency domain. After we decomposed the original data series into many sinusoidal components, probably we will easily detect an energy content that corresponds to the rotational frequency. The place of this peak on the X-axis means the main operational frequency. For understanding all of the detected frequency peaks we have to know exactly how the analyzed assembly works. In case of a simpler rotary machine we are able to compute the rotating speed directly from spectral result. On the other hand for example in case of an internal combustion engine the activity of cylinders gives the dominant frequency. Before frequency analysis it is necessary to clarify which part of assembly will be the dominant vibration source.

The equation that describes rotating speed in dimension of

RPM (Revolutions Per Minute) is as follows:

$$n = 60 \cdot f_1 \tag{9}$$

where *n* is the rotating speed and f_1 shows the main frequency in Hz.

The engine speed can be worked out according to the first main harmonic order when the number of cylinders is known. Assuming $f_{i/2}$ is the frequency of the first main harmonic order k=i/2, f_1 is the rotating frequency of the engine, the engine speed *n* will be:

$$n = 60 \cdot f_{i/2} / k = 120 f_{i/2} / i$$
[3]
(10)

A. Spectrum analysis

Maybe the most commonly used method in this category is the so called Auto Power Spectrum (APS) calculation. To compute the APS, the FFT (Fast Fourier Transformation) of the signal is computed, then multiplied by its complex conjugate. Hence the magnitude of an APS is equal to the square magnitude of an FFT. FFT is practically equivalent to the Linear Spectrum.

This is one methodology, but APS can be calculated at several ways. The PSD (Power Spectral Density) is the PSD normalized to a 1 Hz bandwidth. That means PSD is the APS divided by the interval between frequency data points. The PSD can be produced as the Fourier transformation of the ACF.

The theory of Fourier transformation was originally developed for continuous signals. The equation for Power Spectrum calculation for this signal type is the following:

$$P(\omega) = \left| \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt \right|^2 = X(\omega) X^{\bullet}(\omega)$$
(11)

$$X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t} dt \to Fourier$$
(12)

$$\omega = 2\pi f \tag{13}$$

Of course it is possible to expand this theory for discrete time signals as well. The equation can be written as follows:

$$F(x[n]) = X[n] = \Delta t \sum_{k=0}^{N-1} x[k] e^{-j\frac{2\pi}{N}kn}$$
(14)



Auto Power Spectrum Fig. 6. APS of acceleration signal

By spectral analysis in most cases we cut signal firstly into smaller pieces (overlap and windowing can be applied). After spectral calculation is done for every separated piece of data series, it is possible to make conclusions based on the averaged result. Of course spectral analysis can be calculated without averaging, but it is an effective method to eliminate the effect of random noises and to get a smoother output. In the following study 8192 wide Hanning window was used to compute the averaged spectra. For the resulting plot, see Fig. 6.

As can be seen from Fig. 6., the method gives us a representative result. After the DC component (0 Hz) the first prominent spectrum peak belongs to the base harmonics of vibration. The location of this peak on the frequency axis is the rotational frequency. The spectral analysis gives us a result with linear frequency resolution. In this case the measurement data series has got a sampling rate of 51.2 kHz, the spectrum was calculated with 8192 samples long window size. These details result a frequency resolution of $\Delta f=6.25$ Hz. The location of the peak is at 75Hz that means a rotational speed of 4500 RPM. However the resolution causes 8.3% relative uncertainty. The reliability becomes better by higher speed.

Spectrum based estimation is possible based on upper harmonics as well. Accordingly the location of a specific harmonic component can be defined with better relative accuracy. From this frequency value the natural frequency can be calculated by a simple division. The advantage of the upper harmonic based method is the much lower reading error, however this method has got its disadvantages as well. As higher the upper harmonic's frequency as lower the SNR (Signal to Noise Ratio). It results problematic harmonic peak detection. The measured noise type can consider as white noise. It means that every frequency component is equally contained regarding the energy of noise. In spite of this the energy level of vibration components from mechanical operation is as lower as higher frequency is examined. The reasonable consequence of the previously described facts is the increasingly low SNR.

B. Analysis by Short Time Furier Transformation

In another method the spectra calculated for the Fig. 7. Result of STFT windowed signal sections is used not for averaging. This partial results can be used also for analyzing the

spectral changes over time. This consideration brings us to the Short Time Fourier Transformation (STFT). Equation (15) demonstrate the calculating method of STFT for sampled time signals.

$$STFT\{x[n]\}(m,\omega) \equiv X(m,\omega) = \sum_{m=-\infty}^{\infty} (x[n]w[n-m]e^{-j\omega n})^{(15)}$$

The resolution of the result is determined by the sampling frequency and the block size used for FFT calculation. The disadvantage of the method is that for want of averaging the result is noisier than the traditional spectral calculation's output. Nevertheless the great advantage of STFT is that it gives us evaluable result by variable rotational speed as well. In this case the averaged spectrum produces smeared peaks which can be more distorted because of the moving upper harmonics and other noises. The interpretability of the result obtained by STFT depends always on a compromise.

By large window size we get high frequency • resolution. Of course as larger the frequency







Fig. 8. STFT representation after image processing

resolution is, as accurate the rotational speed can be determined from time-frequency representation.

By using small window size the resolution along the axis became lower, however the frequency localizability in time will be better. It becomes possible to observe or analyze changing state processes due to the better resolution in time.

The output of STFT is a two-variable distribution whose domain is the two-dimensional (t, f) space. Its constant t cross-section should show the frequency or frequencies present at time t, and its constant f cross-section should show the time or times at which frequency f is present. We can display this two-variable function as an image and also post-process this result with common image processing methods. Thus we can detect ridges, edges or apply digital convolution filters. The output of ridge detection is shown in Fig. 8. The first ridge is at 75 Hz, which equals to 4500 RPM rotational speed. The available resolution with same window size is as in the further presented averaged PSD method. [7]

C. Rotational speed estimation via Cepstrum analysis

The cepstrum was originally defined as the power spectrum of the logarithm of the power spectrum. Later, a newer definition was coined as the inverse transform of the logarithm of the power spectrum or mathematically:

$$C(t) = FFT^{-1}(\ln|FFT(x(t))|)$$
(16)

In the literature four basic kinds of cepstral representations can be found. These are the real-, complex-, power- and phase cepstrum.

Real cepstrum:

$$c(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \log |X(\omega)| e^{j\omega t} d\omega$$
⁽¹⁷⁾

- Complex cepstrum: $\vec{c}(t) = \frac{1}{2\pi} \int_{0}^{\pi} \log(X(\omega)) e^{j\omega t} d\omega$ (18)
- Power cepstrum:

$$c(t)^{2} = \frac{1}{2\pi} \left| \int_{-\pi}^{\pi} \log \left| X(\omega) \right| e^{j\omega t} d\omega \right|^{2}$$
(19)

Fourier phase cepstrum:

$$c_F(t) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \arctan\left(\frac{\Im(X(\omega))}{\Re(X(\omega))}\right) e^{j\omega t} d\omega$$
(20)

More detailed studies about the history and applications of cepstrum analysis and further mathematical background can be found in the following references: [8][9][10][11]

Fig. 8. represents the real cepstrum of the analyzed data series. The illustration shows the outputs calculated for all 3 channels of the 3D accelerometer.

This method converts the signal into quefrency components. One of the earliest applications of the cepstrum was in the study of signals containing echoes and in the study of speech



Fig. 8. Result of Cepstrum analysis



Fig. 9. Result of TCA

analysis. The discussed applications in these themes were aimed at detecting the harmonic structure of measured sound. Such harmonic structures can be detected by gearbox analysis or by any analysis of rotary machines as well. The cepstrum calculation of a signal has got evaluable result only if it contains spectral harmonics. It gives a periodic structure to the spectrum. For example a clear one-component sinusoidal signal can be examined well by power spectrum, however its cepstra shows us nothing useful. The cepstrum analysis searches the families of sidebands in the spectra and shows the dominancy of it.

As it was shown previously by spectral analysis, we also should make windowing by cepstral calculation. After that we should also make averaging for the partial results in the same way. The presented result (Fig. 8.) was made with the same parameters as by the power spectrum calculation. That means 8196 sample was the width of the used Hanning window. In this case the first dominant peak after the noisy section close to 0 quefrency belongs to the rotational speed or the so called main harmonic component. The quefrency location of this peak shows the period time of one full revolution in dimension of seconds. The peak can found at 0.0133s according to the quefrency axis and the data series has got the same resolution in quefrency as the source signal had got in time. It means a quefrency step size of 0.0195ms according to the applied 51.2 kHz sampling rate. These two properties result a relative accuracy of 0.146% by rotational speed estimation. The method gives the expected 4500 RPM rotating speed.

By lower quefrencies the result is usually much noisier. That is why this rotational speed estimating method become unreliable at very high speeds (small rotating period time).

D. Rotational speed estimation by Time-Cepstrum Analysis

The relationship between APS and STFT was discussed in the previous chapters. By analogy we can define an algorithm to examine the cepstrum changes in time. For this method sections from time signal are windowed and cepstrum is computed based on these pieces. If we use this partial results not for averaging, it is possible to represent the time varying TCA (Time-Cepstrum Analysis) in 3 dimensional graphs.

A great advantage of cepstrum and TCA as well is that the resolution along the quefrency axis is not influenced by the window size which is used for the analysis. It depends just from the sampling rate of the source signal. However specifying the suitable block size is not a clearly definable task. If we choose a critically short section the period time should fall out of the result's quefrency range. Another problem can be if the spectra of the examined section gives us not enough information about the upper harmonics of the main frequency. In this cases the TCA produces a problematically evaluable output. Increasing the applied window size effects negatively for the localizability of events in the time signal. The same mechanisms of action was discussed in the case of STFT.

The TCA method gives the same result by rotating speed estimation as the averaged cepstrum output. It means we get the expected 4500 RPM rotating speed in the examined steady state section with prominent relative accuracy.

VI. SUMMARY

The presented methods for rotating speed estimation pose a great convenience if the examined rotary machine hasn't got any built in sensor for rotating speed measuring purpose. It is possible to complete the assembly with other sensors, but it causes often difficulty because of the complexity of devices. Furthermore the gross error of a widely used solution with optical sensor was tested as well. For such a problem many of vibration diagnostic methods can supply solution. The easiest way is to determine the periodicity of vibration signal directly in the time domain. The usability of signal comparison is better after signal conditioning. We can also determine periodicity from the calculated autocorrelation function. There are several methods to transfer signal into other domains by spectra or cepstra calculation. For these methods it is needed to cut out a specified length of the original time signal and transform this section at once. The relative accuracy by rotational speed estimation depends on the method, the applied block size, the sampling rate of time signal and the actual rotational speed. All of the presented methods are able to show us the main parameters of operation, but the spectral analysis has got as good relative resolution as high the rotational frequency is. However the cepstrum have got an inverse behavior. In case of both methods the SNR became worse at high speeds. There are available methods for estimation by varying rotational speed as well, these are the discussed STFT and the TCA. These solutions have got almost the same properties about relative accuracy as the spectrum or the cepstrum calculations.

It is possible to develop such hybrid methods for digital signal processing, which can combine the above discussed calculations. The ground of the hybrid calculation should switched based on a survey of signal components, noise level, basis frequency etc. The relative accuracy of several methods can be estimated from the result of previous iterations and the hybrid logic should make decision, which theory supports the best result.

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